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Physical Description of the Single-Mode and Multimode Fiber Channels

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Outline

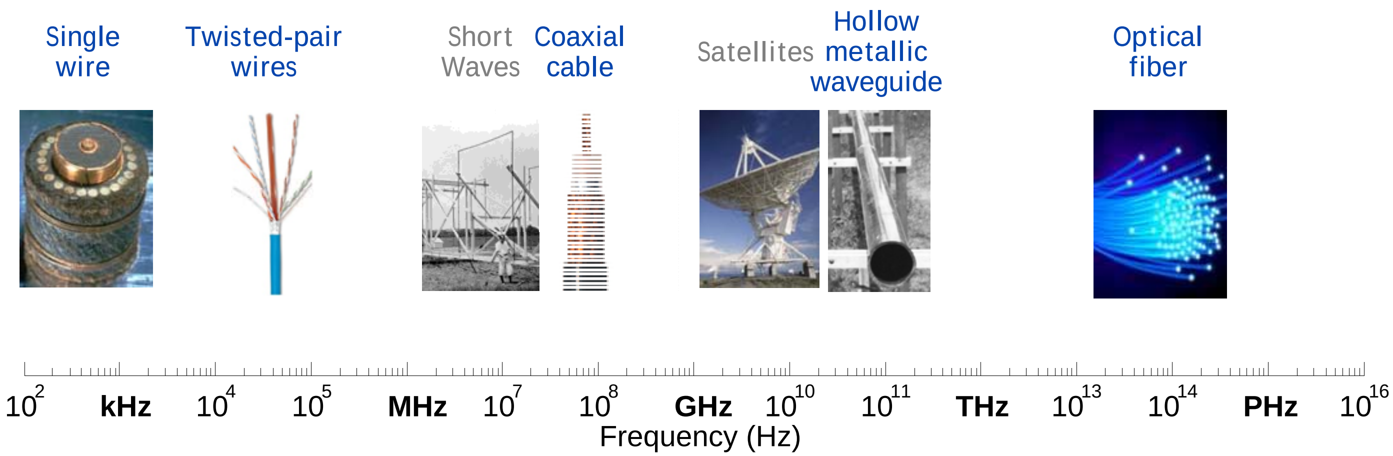
1. Introduction
2. Single-mode fiber (SMF)
3. Nonlinear Shannon limit for SMF
4. Space-division multiplexing (SDM) fibers
5. MIMO in fiber-optic systems
6. Outlook

$>$ 99% of long-distance data traffic goes through optical fibers

What is the theoretical capacity of optical fibers?

Challenge comes from that an **optical fiber is a nonlinear medium**

Wired Communication and the Electromagnetic Spectrum



There has been an increase by more than 12 orders of magnitude in bandwidth for wired transmission

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What Does It Take to Connect the World

- Humans on Earth, $N_{humans} \sim 10^{10}$
- Communication rate $R_{bit} \sim 1$ Gbit/s
- The total data rate needed:

$$R_{total} \sim N_{humans} R_b$$

$$= 10^{10} \times 10^{9} = \mathbf{10^{19}\ bit/s}$$

or

100 billions times a typical home data rate (100 Mbit/s)

Requires large bandwidth channel and spatial confinement

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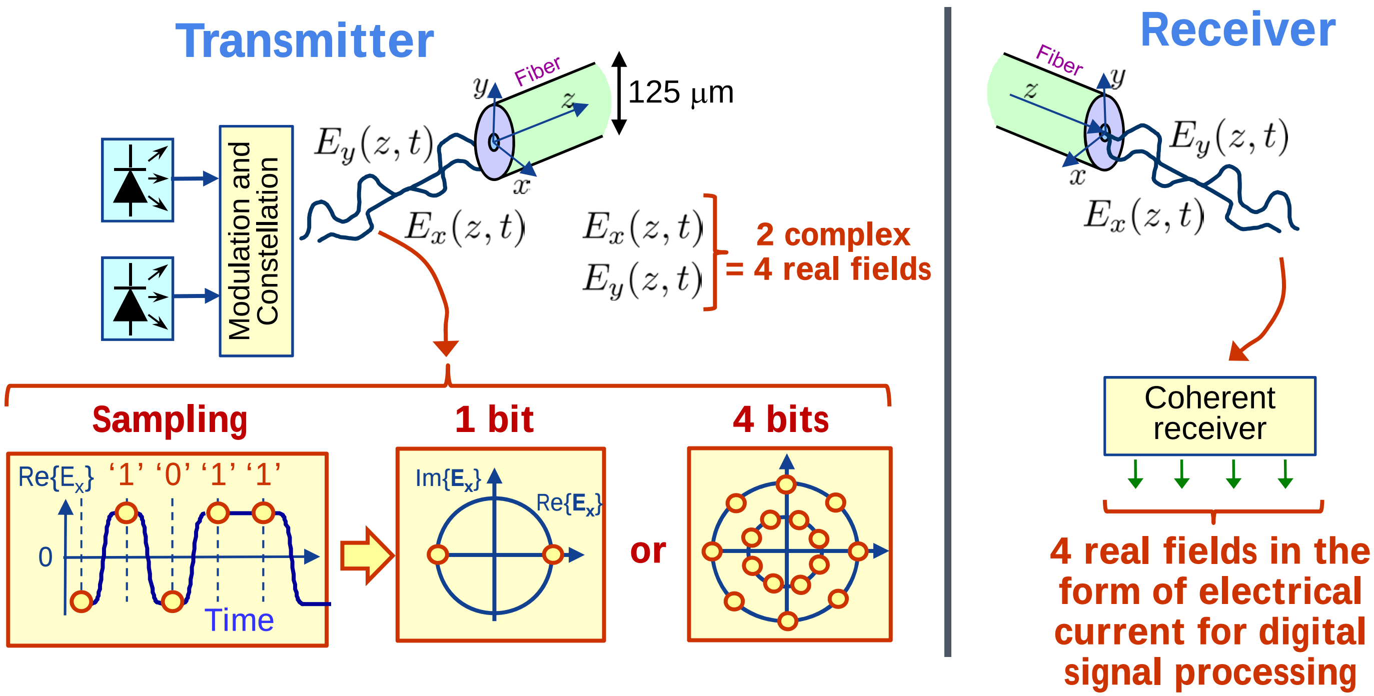
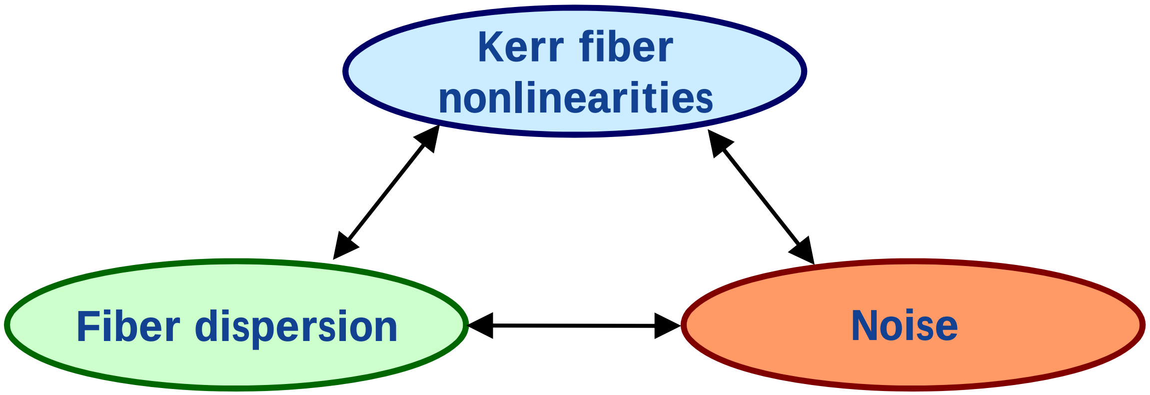
Optical Fiber Transmitter and Receiver



Optical fiber is well suited for ultra-high capacity transport in confined spatial dimensions

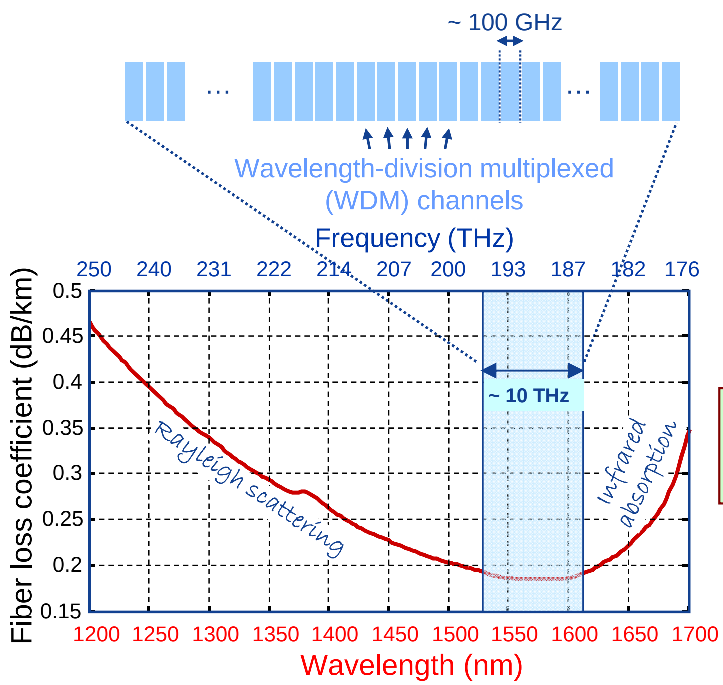
Three Main Physical Phenomena Affecting Fiber Transmission



These effects occur **simultaneously** with an optical fiber

Optical Fiber Bandwidth

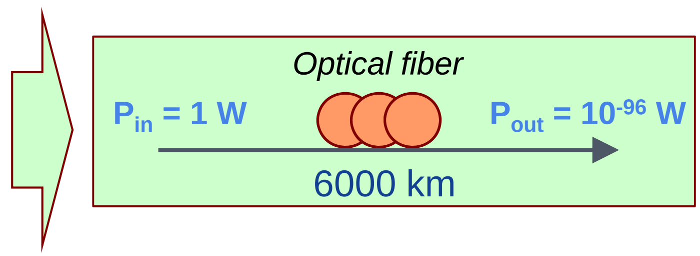
Fiber loss and signal frequencies



Loss over transoceanic distances

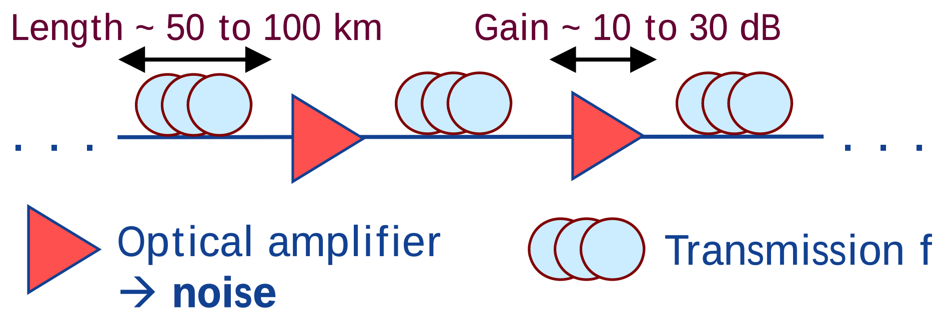
- Fiber loss through the Atlantic Ocean

  0.16 dB/km x 6000 km ~ 960dB



Optical amplifiers are required for long-distance fiber transmission

Optical Amplification in a Optical Fiber Transmission Line



- Optical amplifiers are inserted periodically
- Amplification adds distributed noise
- Signal is loaded with noise when arriving to destination

A transmission line generates **a fixed spectral density of the noise**, independently of signal power

Fiber Chromatic Dispersion

- Chromatic dispersion equation:

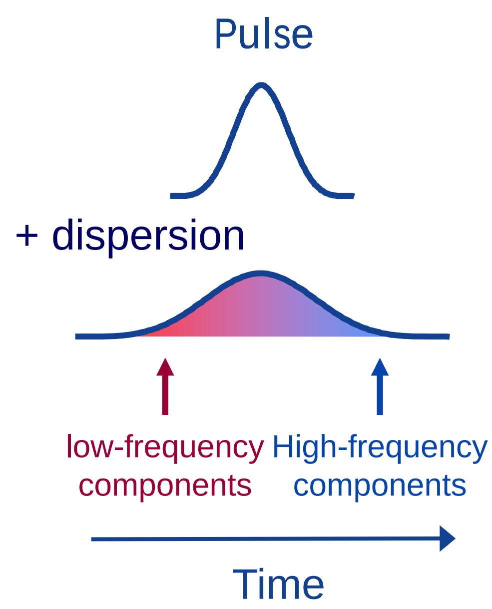
$$\frac{\partial E}{\partial z} + \frac{i}{2}\beta_2 \frac{\partial^2 E}{\partial t^2} = 0$$

- It can be solved in the spectral domain:

$$E(t) = \frac{1}{\sqrt{2\pi}} \int \widetilde{E}(\omega) e^{i\omega t} d\omega$$

$$\frac{d\widetilde{E}(\omega)}{dz} = \frac{i}{2}\beta_2 \omega^2 \widetilde{E}(\omega)$$

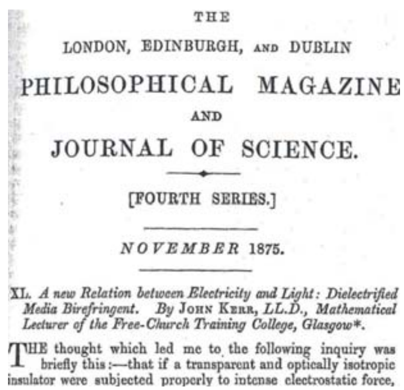
$$\widetilde{E}(z,\omega) = \widetilde{E}(0,\omega)\ \exp(\,i\,\beta_2\,\omega^2\,z\,/\,2)$$



Chromatic dispersion is an all-pass filter

The Optical Kerr Effect (or AC Kerr Effect)

John Kerr
(1824-1907)



THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[FOURTH SERIES.]

NOVEMBER 1875.

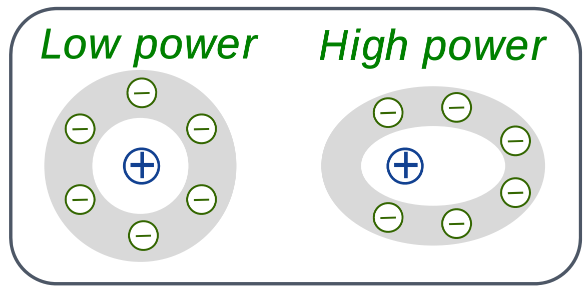
XL. *A new Relation between Electricity and Light: Dielectrified Media Birefringent.* By JOHN KERR, *LL.D., Mathematical Lecturer of the Free-Church Training College, Glasgow**.

THE thought which led me to the following inquiry was briefly this:—that if a transparent and optically isotropic insulator were subjected properly to intense electrostatic force,

First paper on the "Kerr effect" (1875)

- The **light** electromagnetic field **distorts the electronic cloud**

Electronic cloud distortion



The signal phase is increasingly distorted by increasing signal power

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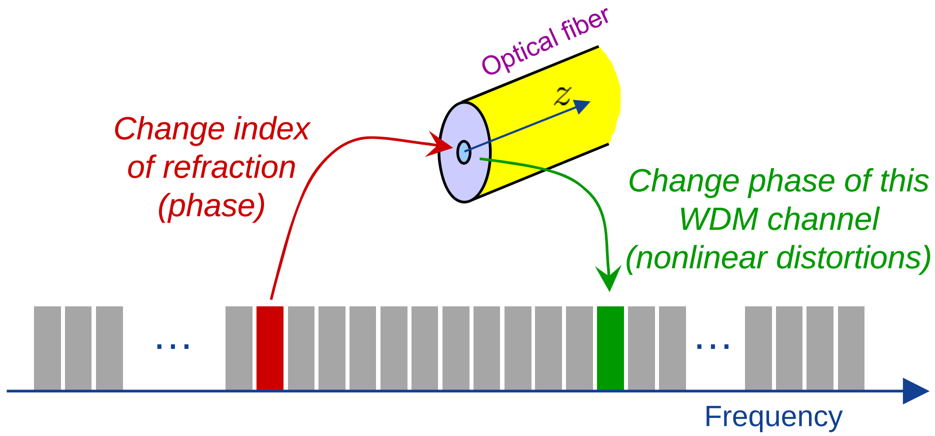
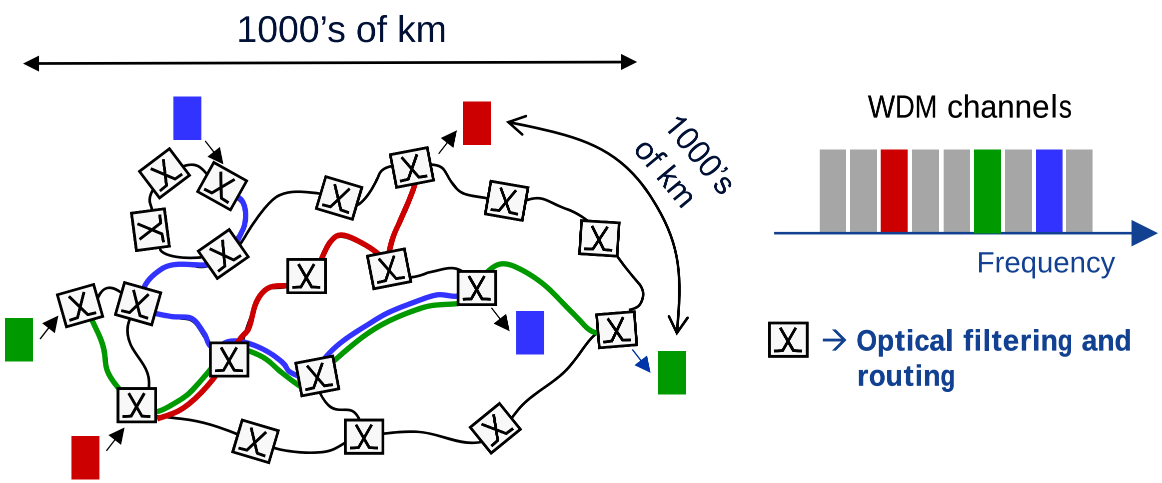
Nonlinear Crosstalk in Optical Fibers



- Signal distortions occur without spectral overlap

The nonlinear response of the glass is the main source of interference between WDM channels

Consequences of Routing in Optical Networks



- Different origins and destinations for different WDM channels
- WDM channels co-propagate over only a portion of the optical path

Joint processing of WDM channels not possible in optically-routed networks

Nonlinear Propagation in Optical Fibers (Single Polarization)

- One wants to solve the evolution of field $E(z,t)$ with distance:

$E(0,t)$ →	→ $E(z,t)$
Input — *Fiber propagation with amplifier noise* —	*Output*

- Need to solve the stochastic nonlinear Schrödinger equation (SNSE):

$$\frac{\partial E(z,t)}{\partial z} + \frac{\imath}{2}\beta_2 \frac{\partial^2 E(z,t)}{\partial t^2} - \imath\,\gamma\,|E(z,t)|^2\,E(z,t) = \imath\,N(z,t)$$

Fiber dispersion (all-pass filter)

Fiber Kerr nonlinearities (include all WDM channels)

Noise (AWGN)

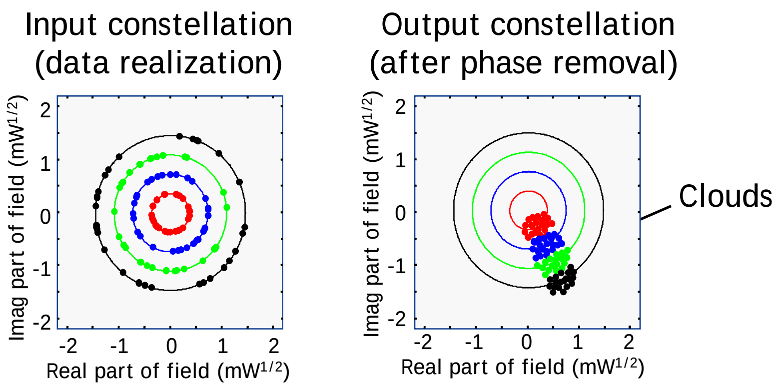
- In the absence of nonlinearities → capacity of the AWGN channel
- At high powers, the nonlinear term dominates

No exact general analytical solution to the SNSE exists

AWGN: Additive white Gaussian noise

Calculation of Optical Fiber Capacity Estimate

- Numerical solution of the SNSE to capture all nonlinear effects
- Optimization of:
  1. Optical amplification (distributed)
  2. Digital signal processing (digital back-propagation)
- Ring constellations



- Fit a probability density functions (PDFs) to clouds
- Calculate mutual information from PDFs

It leads to a nonlinear capacity limit estimate

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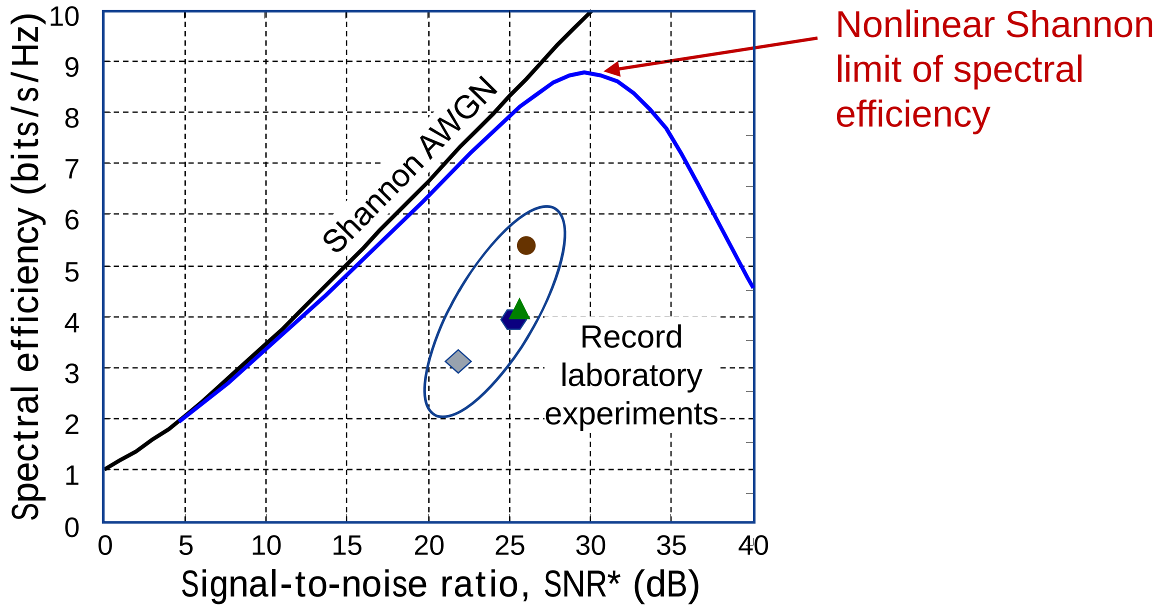
Nonlinear Shannon Limit Estimate (Single Polarization)

500 km of standard single-mode fiber (SSMF)



**SNR: signal average power in the fiber divided by the AWGN power per symbol*

Experimentally demonstrated spectral efficiencies have leveled off to ~60% of maximum spectral efficiency since 2012

Nonlinear Shannon Limit Formula

An analytical formula for the capacity per unit of bandwidth *C*:

$$C = \log_2\left(1 + \left[\frac{n_{\mathrm{sp}}\hbar\omega_0\alpha L R_s}{P_0} + 4\frac{\gamma^2\mathcal{P}^2 L}{R_s^2|\beta_2|}\right]^{-1}\right)$$

$$\text{where } \mathcal{P} = \left[\sum_{n=N_{\mathrm{left}}(n\neq 0)}^{N_{\mathrm{right}}} \frac{\kappa}{2\pi}\frac{R_s}{|\Delta f_n|}\right]^{1/2} P_0$$

n_{sp}	Spontaneous emission factor
$\hbar\omega_0$	Photon energy
α	Fiber loss coefficient
L	System length
R_s	Symbol rate
P_0	Signal power
γ	Fiber nonlinear coefficient
β_2	Group-velocity dispersion
N_{ch}	Number of WDM channels
Δf_n	Channel n spacing
κ	Mod. format nonlinear factor

Location of the WDM channel in the spectrum

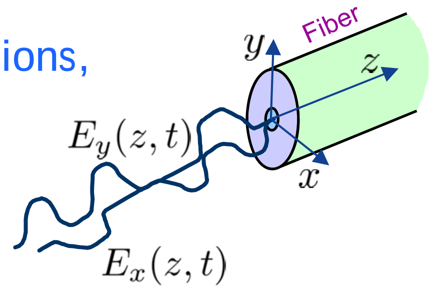
$$\kappa = \frac{n^4/5 + n^3/2 + n^2/3 - 1/30}{(n^2/3 + n/2 + 1/6)^2} - 1$$

n: number of rings

Capacity formula fits reasonably well current numerical capacity limits

Nonlinear Propagation in Optical Fibers (Dual Polarization)

- Nonlinear propagation when two polarizations, $E_x(z,t)$ and $E_y(z,t)$ co-propagate



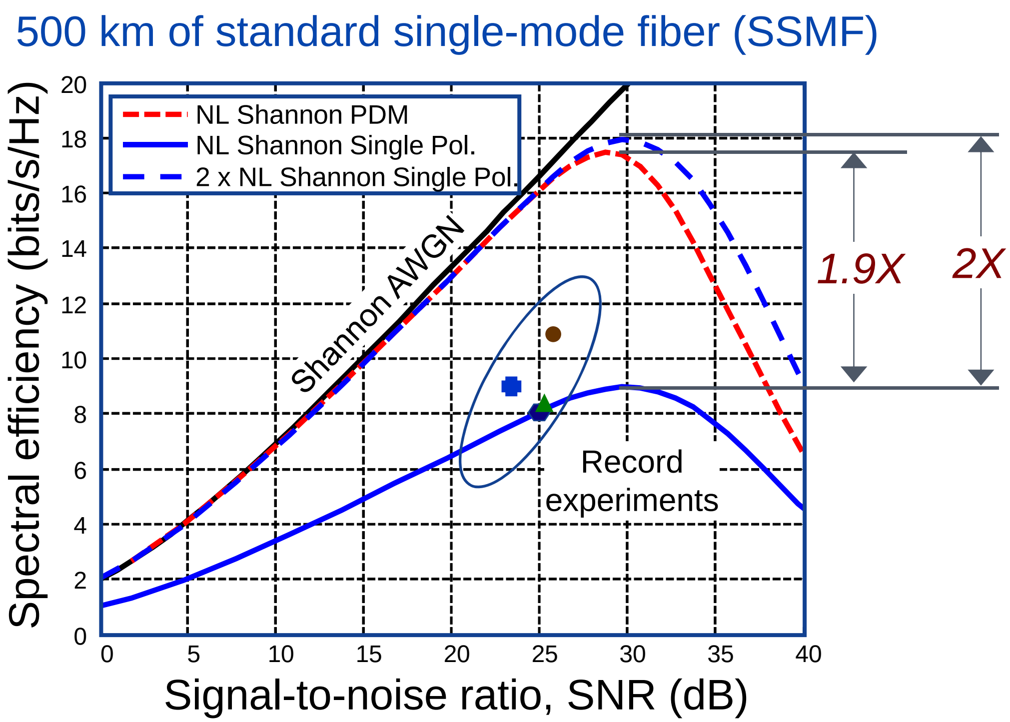
Cross-polarization nonlinearity
(nonlinear crosstalk)

$$\frac{\partial E_x}{\partial z} + \frac{\imath}{2}\beta_2 \frac{\partial^2 E_x}{\partial t^2} - \imath \frac{8}{9}\gamma \left(|E_x|^2 + |E_y|^2\right) E_x = \imath N_x(z,t)$$

$$\frac{\partial E_y}{\partial z} + \frac{\imath}{2}\beta_2 \frac{\partial^2 E_y}{\partial t^2} - \imath \frac{8}{9}\gamma \left(|E_y|^2 + |E_x|^2\right) E_y = \imath N_y(z,t)$$

Nonlinear coupling between polarizations impacts transmission

Nonlinear Shannon Limit Estimate for Polarization-Division Multiplexing (PDM)



Transmitting two polarizations nearly doubles fiber capacity but not quite

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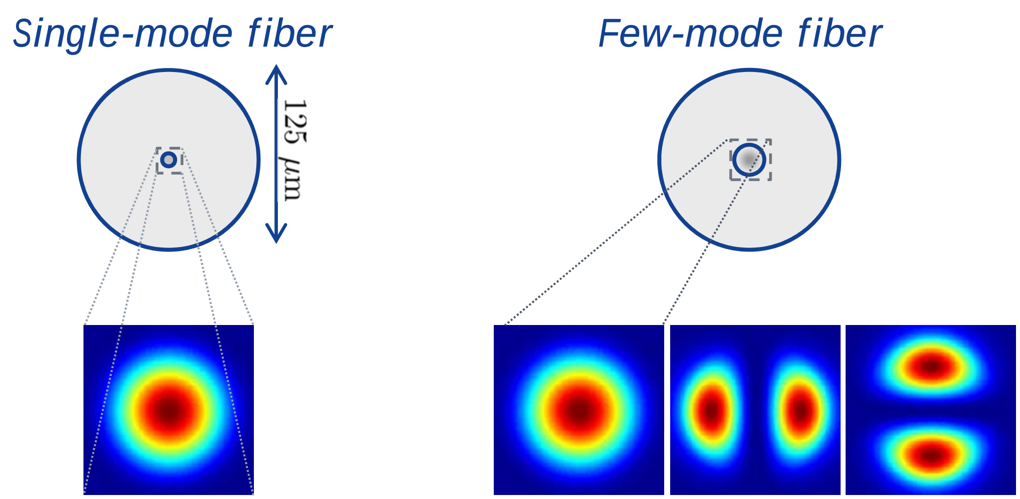
A Single Fiber Strand Can Support Multiple Spatial Modes

Fiber cross-sections



1 spatial mode x 2 pol.
= 2 modes

3 spatial modes x 2 polarizations
= 6 modes

One can design different optical fibers (“engineer the channel”)

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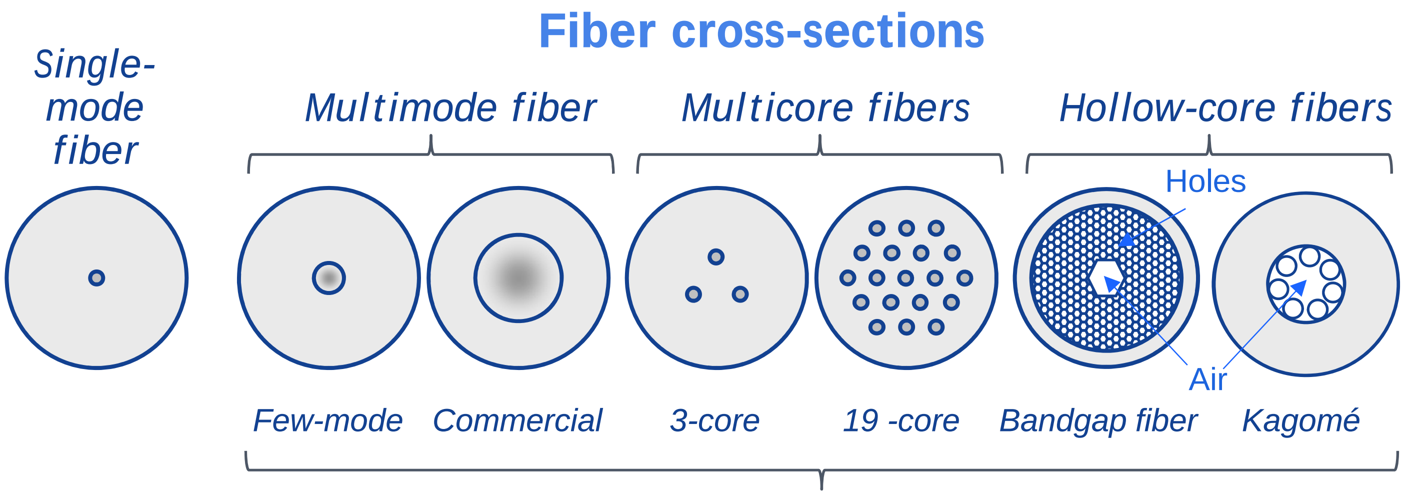
Single Fiber Strands Supporting Multiple Spatial Modes



- These fibers enables **space-division multiplexing (SDM)** in fibers
- Most require **multiple-input multiple output (MIMO)** processing
- **Capacity per fiber strand can increase dramatically**

Can these advanced technologies reduce the cost per bit transported?

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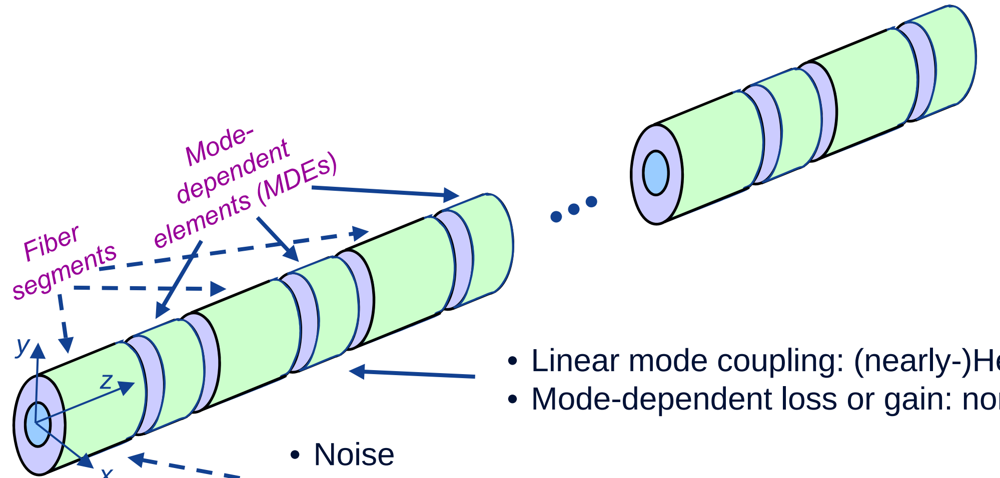
Modelization of Propagation Effects in SDM Fibers

- A space-division multiplexing (SDM) fiber can be viewed as:



- Linear mode coupling: (nearly-)Hermitian
- Mode-dependent loss or gain: non-Hermitian

- Noise
- Chromatic dispersion
- Kerr Nonlinearity

- The *fiber segments* model propagation without linear compling
- The *mode-dependent elements* (MDEs) introduce linear mode mixing (coupling) and mode-dependent

Linear coupling arises from “imperfect” fibers and lead to the need for MIMO processing

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MIMO in Fiber Channel

In single-mode fibers:

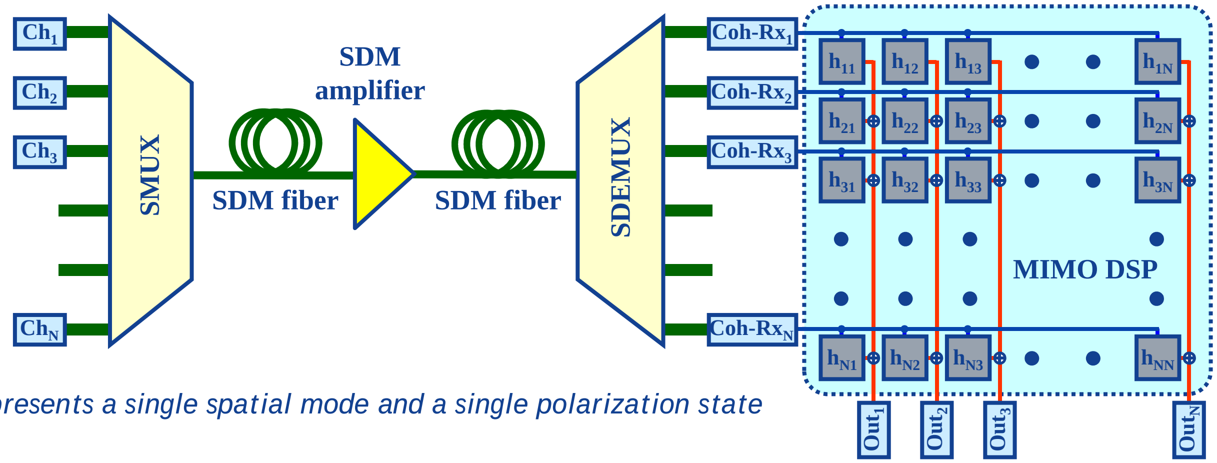
- In single-mode fibers: 2x2 MIMO is done since 2005
- Up to about 20 symbols need to be processed simultaneously
- Current speeeds are
  a. 1 Tb/s (offline post-processing)
  b. 400 Gb/s (real-time)

In space-division multiplexed (SDM) fibers:

- Up to 12x12 MIMO has been demonstrated
- Up to 1000 symbols need to be processed simultaneously
- Current speeds are a few Tb/s (offline post-processing)

Mode-dependent loss/gain and nonlinear effects degrade the efficiency of MIMO

Schematic of Coherent MIMO-based Coherent Crosstalk Suppression for Space-Division Multiplexing (SDM)



Represents a single spatial mode and a single polarization state

- All guided modes of the SDM fiber are selectively launched
- All guided modes are linearly coupled during propagation in the SDM fiber
- All guided modes are simultaneously detected with coherent receivers
- Multiple-input multiple-output (MIMO) digital signal processing decouples the received signals to recover the transmitted signal

Crosstalk from spatial multiplexing can be nearly completely removed by MIMO digital signal processing

Adapted from Morioka et al., IEEE Commun. Mag., pp. S31-S42 (2012)

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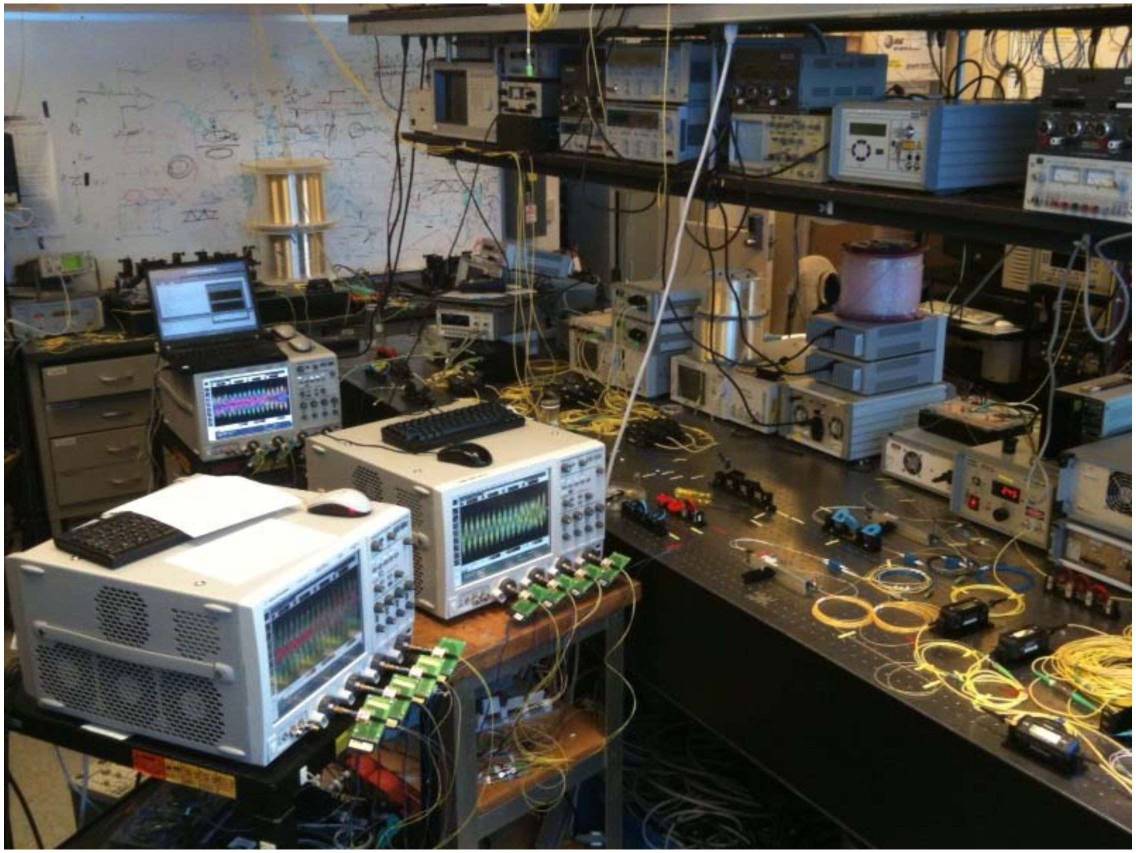
6 x 6 coherent MIMO experiment



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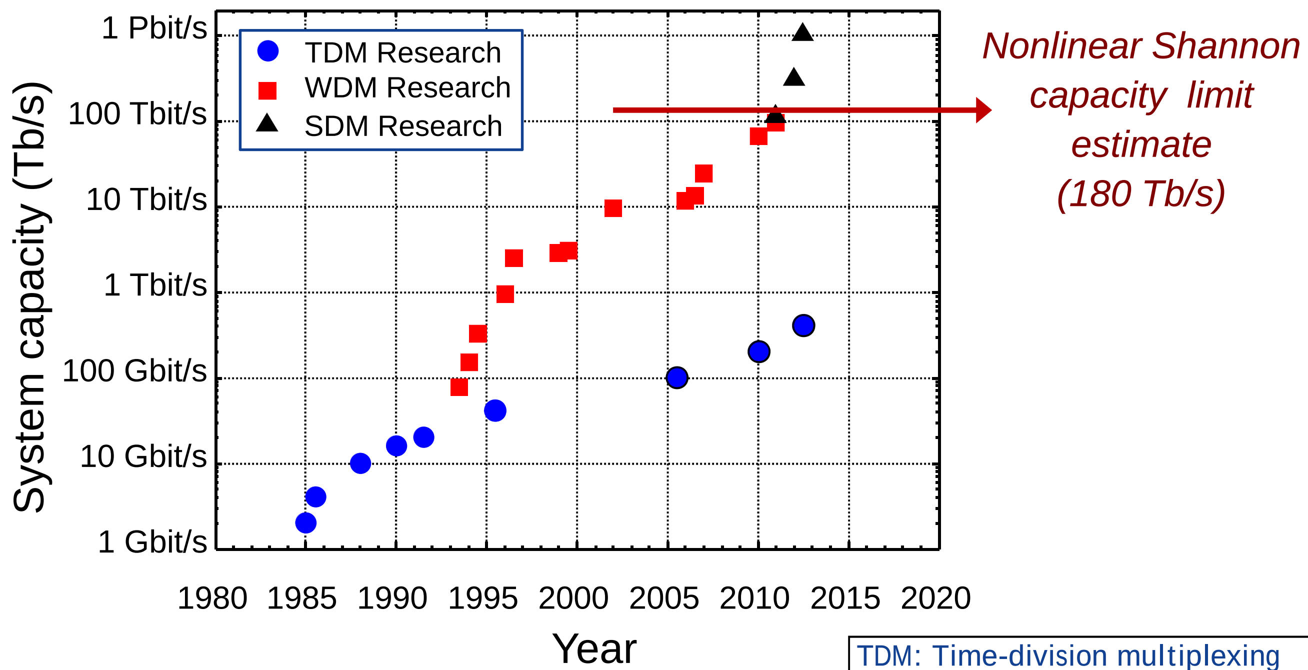
Historical Capacity Evolution



TDM: Time-division multiplexing
WDM: Wavelength-division multiplexing
SDM: Space-division multiplexing

Space-division multiplexing has already exceeded the nonlinear Shannon limit

Summary and Outlook

- **Nonlinear Shannon capacity limit has been estimated**
  → ~100 Tb/s over 1000 km → 100,000 fibers for 10^{19} bit/s
- **Space-division multiplexing is a promising technology**
  to further increase capacity per fiber strand
- **In the absence of mode-dependent loss or gain, MIMO is about finding a unitary matrix**

The "fiber channel" is either rather simple or exceedingly complex depending on the propagation effects included

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** by alphabetic order*

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