Physical Description of the Single-Mode and Multimode Fiber Channels

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Outline

1. Introduction
2. Single-mode fiber (SMF)
3. Nonlinear Shannon limit for SMF
4. Space-division multiplexing (SDM) fibers
5. MIMO in fiber-optic systems
6. Outlook
> 99% of long-distance data traffic goes through optical fibers

What is the theoretical capacity of optical fibers?

Challenge comes from that an optical fiber is a nonlinear medium
There has been an increase by more than 12 orders of magnitude in bandwidth for wired transmission.
What Does It Take to Connect the World

- Humans on Earth, \( N_{\text{humans}} \sim 10^{10} \)
- Communication rate \( R_{\text{bit}} \sim 1 \text{ Gbit/s} \)
- The total data rate needed:
  \[
  R_{\text{total}} \sim N_{\text{humans}} R_{b} = 10^{10} \times 10^{9} = 10^{19} \text{ bit/s}
  \]
  or
  
  100 billions times a typical home data rate (100 Mbit/s)

Requires large bandwidth channel and spatial confinement
Optical Fiber Transmitter and Receiver

**Transmitter**

Modulation and Constellation

\[
\begin{align*}
E_y(z, t) & \\
E_x(z, t) & \\
E_x(z, t) & \\
E_y(z, t) & \\
\end{align*}
\]

Sampling

1 bit

or

4 bits

\[
\begin{align*}
\text{Re}\{E_x\} & \quad \text{Im}\{E_x\} \\
0 & \quad \text{Time} \\
\end{align*}
\]

Optical fiber is well suited for ultra-high capacity transport in confined spatial dimensions

**Receiver**

Coherent receiver

\[
\begin{align*}
E_y(z, t) \\
E_x(z, t) \\
\end{align*}
\]

4 real fields in the form of electrical current for digital signal processing
Three Main Physical Phenomena Affecting Fiber Transmission

- Kerr fiber nonlinearities
- Fiber dispersion
- Noise

These effects occur simultaneously with an optical fiber
Optical Fiber Bandwidth

Fiber loss and signal frequencies

Loss over transoceanic distances

- Fiber loss through the Atlantic Ocean
  \[0.16 \text{ dB/km} \times 6000 \text{ km} \sim 960 \text{ dB}\]

Optical amplifiers are required for long-distance fiber transmission
Optical Amplification in a Optical Fiber Transmission Line

A transmission line generates a fixed spectral density of the noise, independently of signal power.

- Optical amplifiers are inserted periodically.
- Amplification adds distributed noise.
- Signal is loaded with noise when arriving to destination.
Fiber Chromatic Dispersion

- Chromatic dispersion equation:
  \[
  \frac{\partial E}{\partial z} + \frac{i}{2} \beta_2 \frac{\partial^2 E}{\partial t^2} = 0
  \]

- It can be solved in the spectral domain:
  \[
  E(t) = \frac{1}{\sqrt{2\pi}} \int \tilde{E}(\omega) e^{i\omega t} d\omega
  \]
  \[
  \frac{d\tilde{E}(\omega)}{dz} = \frac{i}{2} \beta_2 \omega^2 \tilde{E}(\omega)
  \]
  \[
  \tilde{E}(z, \omega) = \tilde{E}(0, \omega) \exp\left(\frac{i \beta_2 \omega^2 z}{2}\right)
  \]

Chromatic dispersion is an all-pass filter
The Optical Kerr Effect (or AC Kerr Effect)

• The light electromagnetic field distorts the electronic cloud

Electronic cloud distortion

Low power

High power

The signal phase is increasingly distorted by increasing signal power

John Kerr (1824-1907)

First paper on the "Kerr effect" (1875)
Nonlinear Crosstalk in Optical Fibers

- Signal distortions occur without spectral overlap

The nonlinear response of the glass is the main source of interference between WDM channels
Consequences of Routing in Optical Networks

- Different origins and destinations for different WDM channels
- WDM channels co-propagate over only a portion of the optical path

Joint processing of WDM channels not possible in optically-routed networks
Nonlinear Propagation in Optical Fibers (Single Polarization)

- One wants to solve the evolution of field $E(z, t)$ with distance:

\[
\frac{\partial E(z, t)}{\partial z} + \frac{\gamma}{2} \beta_2 \frac{\partial^2 E(z, t)}{\partial t^2} - \nu \gamma |E(z, t)|^2 E(z, t) = \nu N(z, t)
\]

- Need to solve the stochastic nonlinear Schrödinger equation (SNSE):

- In the absence of nonlinearities $\rightarrow$ capacity of the AWGN channel
- At high powers, the nonlinear term dominates

No exact general analytical solution to the SNSE exists

AWGN: Additive white Gaussian noise
Calculation of Optical Fiber Capacity Estimate

- Numerical solution of the SNSE to capture all nonlinear effects
- Optimization of:
  1. Optical amplification (distributed)
  2. Digital signal processing (digital back-propagation)
- Ring constellations

- Fit a probability density functions (PDFs) to clouds
- Calculate mutual information from PDFs

It leads to a nonlinear capacity limit estimate
Nonlinear Shannon Limit Estimate (Single Polarization)

500 km of standard single-mode fiber (SSMF)

*SNR: signal average power in the fiber divided by the AWGN power per symbol

Experimentally demonstrated spectral efficiencies have leveled off to ~60% of maximum spectral efficiency since 2012
Nonlinear Shannon Limit Formula

An analytical formula for the capacity per unit of bandwidth $C$:

$$C = \log_2 \left( 1 + \left[ \frac{n_{sp} \hbar \omega_0 \alpha L R_s}{P_0} + 4 \frac{\gamma^2 P^2 L}{R_s^2 |\beta_2|} \right]^{-1} \right)$$

where $P = \left[ \sum_{n=N_{\text{left}}(n \neq 0)}^{N_{\text{right}}} \frac{\kappa}{2\pi |\Delta f_n|} \right]^{1/2} P_0$

Location of the WDM channel in the spectrum

Capacity formula fits reasonably well current numerical capacity limits
Nonlinear Propagation in Optical Fibers (Dual Polarization)

- Nonlinear propagation when two polarizations, $E_x(z,t)$ and $E_y(z,t)$ co-propagate

Cross-polarization nonlinearity
(nonlinear crosstalk)

\[
\frac{\partial E_x}{\partial z} + \frac{i}{2} \beta_2 \frac{\partial^2 E_x}{\partial t^2} - \frac{i}{9} \gamma (|E_x|^2 + |E_y|^2) E_x = i N_x(z,t)\\
\frac{\partial E_y}{\partial z} + \frac{i}{2} \beta_2 \frac{\partial^2 E_y}{\partial t^2} - \frac{i}{9} \gamma (|E_y|^2 + |E_x|^2) E_y = i N_y(z,t)
\]

Nonlinear coupling between polarizations impacts transmission
Nonlinear Shannon Limit Estimate for Polarization-Division Multiplexing (PDM)

500 km of standard single-mode fiber (SSMF)

*Transmitting two polarizations nearly doubles fiber capacity but not quite*
A Single Fiber Strand Can Support Multiple Spatial Modes

Fiber cross-sections

**Single-mode fiber**
- 1 spatial mode x 2 pol.
  = 2 modes

**Few-mode fiber**
- 3 spatial modes x 2 polarizations
  = 6 modes

One can design different optical fibers (“engineer the channel”)
Single Fiber Strands Supporting Multiple Spatial Modes

Fiber cross-sections

- Single-mode fiber
- Multimode fiber
  - Few-mode
  - Commercial
- Multicore fibers
  - 3-core
  - 19-core
- Hollow-core fibers
  - Bandgap fiber
  - Kagomé

• These fibers enable space-division multiplexing (SDM) in fibers
• Most require multiple-input multiple output (MIMO) processing
• Capacity per fiber strand can increase dramatically

Can these advanced technologies reduce the cost per bit transported?
Modelization of Propagation Effects in SDM Fibers

- A space-division multiplexing (SDM) fiber can be viewed as:

  - The fiber segments model propagation without linear coupling
  - The mode-dependent elements (MDEs) introduce linear mode mixing (coupling) and mode-dependent effects

Linear coupling arises from “imperfect” fibers and lead to the need for MIMO processing
MIMO in Fiber Channel

In single-mode fibers:

- In single-mode fibers: 2x2 MIMO is done since 2005
- Up to about 20 symbols need to be processed simultaneously
- Current speeds are  
  a. 1 Tb/s (offline post-processing)
  b. 400 Gb/s (real-time)

In space-division multiplexed (SDM) fibers:

- Up to 12x12 MIMO has been demonstrated
- Up to 1000 symbols need to be processed simultaneously
- Current speeds are a few Tb/s (offline post-processing)

Mode-dependent loss/gain and nonlinear effects degrade the efficiency of MIMO
Schematic of Coherent MIMO-based Coherent Crosstalk Suppression for Space-Division Multiplexing (SDM)

- All guided modes of the SDM fiber are selectively launched
- All guided modes are linearly coupled during propagation in the SDM fiber
- All guided modes are simultaneously detected with coherent receivers
- Multiple-input multiple-output (MIMO) digital signal processing decouples the received signals to recover the transmitted signal

Crosstalk from spatial multiplexing can be nearly completely removed by MIMO digital signal processing

*Adapted from Morioka et al., IEEE Commun. Mag., pp. 531-542 (2012)*
6 x 6 coherent MIMO experiment
Space-division multiplexing has already exceeded the nonlinear Shannon limit.
Summary and Outlook

- **Nonlinear Shannon capacity limit has been estimated**
  \[ \approx 100 \, \text{Tb/s over 1000 km} \Rightarrow 100,000 \text{ fibers for } 10^{19} \text{ bit/s} \]

- **Space-division multiplexing is a promising technology**
  to further increase capacity per fiber strand

- **In the absence of mode-dependent loss or gain, MIMO**
  is about finding a unitary matrix

The “fiber channel” is either rather simple or exceedingly complex depending on the propagation effects included.
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